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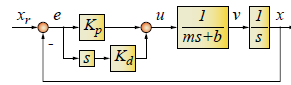
MTE 360 Automatic Control Systems

Laboratory 2: PD/PID Control, Steady State Error and Stability Analysis

## Part 1

1 a)

From the PD controlled servo system block diagram:



Then,

From Lab 1,

1 b)

From the block diagram, let:

The resulting transfer function is

Since, for an ideal 2nd order system, the transfer function is:

Then relating that to the closed loop system above,

From the design specifications:

In addition, recalling the values from 1 a)

1 c)

The ideal system response and the theoretical response of the PD-controlled system are presented below:

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Note that the D-controlled system has an additional left-hand plane pole. This increases the overshoot as can be seen in the graphs. Since zeta = 0.8 is fairly close to 1, the ideal system has very little overshoot, with the system response fairly similar to that of a critically damped system. The addition of the additional zero to allow for D-control creates a much higher degree of overshoot. The pole-zero maps presented below show that the systems have identical pole locations, but the derivative controlled system has an additional zero at Kp / Kd = 36.58.

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1 g)

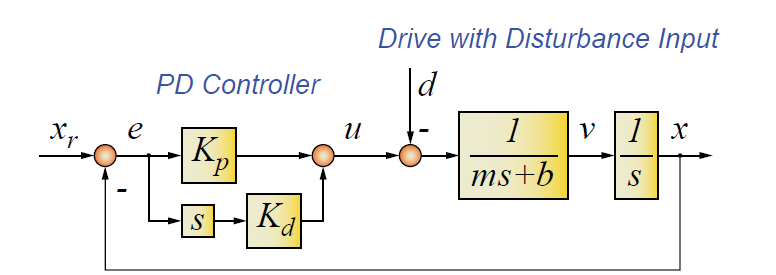
The graphs for simulated and experimental square wave tracking and smooth trajectory tracking for the PD controller are shown below:

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Similarities/discrepancies b/w simulation and experimental results: [please check over]

* During periods where the position is constant, there is a constant 0.1mm tracking error and the system does not try to correct it.
* The tracking error is highest (around +/-0.8mm for smooth trajectory) when the system moves from a period of rest.
* During constant velocity, the tracking error is constant (around 0.6mm for smooth trajectory).

## Part 2



2 a) To express the position as a function of the trajectory command and disturbance:

From the block diagram, let:

Using superposition on PD controlled system with disturbance:

2 b) To express the tracking error as a function of the trajectory command and disturbance:

Using superposition on PD controlled system with disturbance:

2 c) Theoretical expression for steady state error during constant velocity motion:

From Figure 2 in 1g, was observed to be 0.6mm from the tracking error plot during constant velocity between 0.3s and 0.5s.

From 2b,

Due to constant velocity,

Coulomb friction is a represented by a constant value , so

From 1f,

How are the error components are affected by commanded velocity, friction magnitude dc, other parameters, k etc

* As Kp increases, both the error components decrease
* As either as b or v0 increases, increases
* As dc increases, increases.
* The error components are not affected by Kd

2 d) value from part 1f (following error during constant velocity motion)

From 2c,

The average value of Coulomb friction is:

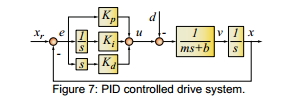
The theoretical value of steady state tracking error in the absence of friction was found in 2c to be:

2 e) rerun with part trajectory from part 1f with friction, overlay sim and exp tracking results (similar to 1g).

* comment on similarity and discrepancy.
* The tracking error varies between 0.7 and -0.4. There is still around 0.2mm of tracking error during periods of constant velocity or position.
* During constant velocity, the tracking error is at a constant 0.6mm for positive change in position and -0.4mm for negative change in position.

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## Part 3



3(a)

Let

and

by inspection

Isolate for x

3(b)

Steady State

3(c) Routh’s Stability Criterion for

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| --- | --- | --- | --- |
|  | 1 |  | 0 |
|  |  |  | 0 |
|  |  | 0 |  |
|  |  | 0 |  |

3(d) Substituting   
 ,, =1.4705

Solving for upper bound of ki:

3 e) How error changes as you modify K? show in graphs similar to part 1g (use 3 diff cases for kp)

* As the value of Ki increases, the average value of tracking error decreases, and the frequency at which tracking error changes from positive to negative increases.
* The frequency at which the system attempts to make corrections increases as Ki increases.
* If Ki > 118, as predicted by the Routh’s stability criterion in 3c, the system becomes unstable (after 1.25s for Ki = 150).
* If Ki < 0, the tracking error will increase to infinity.

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3 f) Show simulation and experimental results, plot similar to 1g.

From Routh’s Stability criterion in 3c,

Using a safety factor of 3,

Using this Ki value, the experimental and simulated results are shown below.

Similarities and differences?

* During periods of constant velocity and periods of constant position, the tracking error is around 0 mm.
* The tracking error is highest (0.4mm) when moving forward from rest at position 0.

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3 g) overlay experimental from parts 1f and 3f. comment on contribution of integral action.

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* Does integral control provide better tracking and disturbance rejection?
  + Integral control provides better tracking as the average value of tracking error is lower
  + It also provides better disturbance rejection as evidenced by 0 tracking error at periods of constant position compared to around 0.1mm for the PD controller.
* What are the limitations
  + At certain values of Ki, the system will be unstable…?
* Refer to 2c, 3b, 3c to back up comments

3 h) plot pole-zero maps of PD and PID transfer functions on the same graph (use diff symbols for each)

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* Comment on pole and zero locations (refer to natural freq and damping ratio values)

By referring to the plot and using Matlab functions: damp() and zero(), the natural frequencies, damping ratios, poles and zeroes were found to be the following:

|  |  |  |  |
| --- | --- | --- | --- |
| **PD Controller** | | | |
| *Freq* | *Zeta* | *Poles* | *Zeroes* |
| 50.265 | 0.8 | -40.212 ± 30.159i | -36.585 |
| **PID Controller** | | | |
| *Freq* | *Zeta* | *Poles* | *Zeroes* |
| 34.489 | 0.340 | -11.741 ± 32.429i | -18.293 ± 25.420i |
| 56.943 | 1 | -56.943 |

* How are closed loop pole locations affected by the integral?