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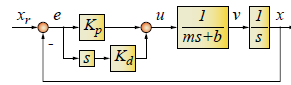
MTE 360 Automatic Control Systems

Laboratory 2: PD/PID Control, Steady State Error and Stability Analysis

## Part 1

1 a)

From the PD controlled servo system block diagram:



Then,

From Lab 1,

1 b)

If the PD controlled system is simplified, the resulting transfer function is

Since, for an ideal 2nd order system, the transfer function is:

Then relating that to the closed loop system above,

From the design specifications:

In addition, recalling the values from 1 a)

1 c)

The ideal system response and the theoretical response of the PD-controlled system are presented below:

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Note that the D-controlled system has an additional left-hand plane pole. This increases the overshoot as can be seen in the graphs. Since zeta = 0.8 is fairly close to 1, the ideal system has very little overshoot, with the system response fairly similar to that of a critically damped system. The addition of the additional zero to allow for D-control creates a much higher degree of overshoot. The pole-zero maps presented below show that the systems have identical pole locations, but the derivative controlled system has an additional zero at Kp / Kd = 36.58.

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1 g)

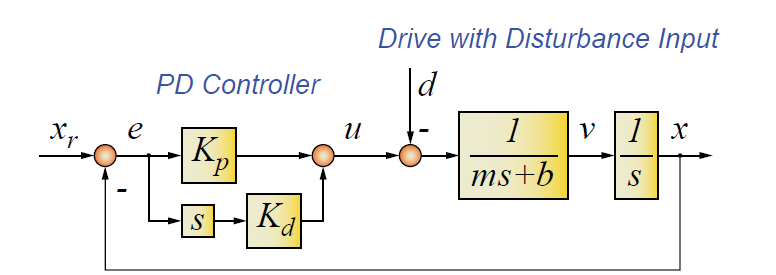
The graphs for simulated and experimental square wave tracking and smooth trajectory tracking for the PD controller are shown below:

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Similarities/discrepancies b/w simulation and experimental results: [please check over]

* Square wave has more discrepancy between simulation and experimental results than the smooth trajectory.
* The square wave experimental results are noisy at each edge for around 0.1 seconds before lowering to a stable value. This is due to the sudden changes in the command positions. The noise the beginning of each edge is caused by the proportional gain which will is high when the difference between the desired position and the current position but decreases as the position is closer to that desired. The overshoot at the beginning go f each edge is caused by the additional pole from the derivative term as mentioned in 1 c).
* The smooth trajectory experimental results are nearly identical to the simulation because of the gradual changes in the position. There is less noise because the proportional gain will be changing in small increments. There is however still a small tracking error as the command position changes.

## Part 2



2 a) To express the position as a function of the trajectory command and disturbance:

Using superposition on PD controlled system with disturbance:

2 b) To express the tracking error as a function of the trajectory command and disturbance:

Using superposition on PD controlled system with disturbance:

2 c) Theoretical expression for steady state error during constant velocity motion:

[From Dan’s calculations, I need some help understanding how to get v0 in there, and how we got 0.6]

* How are the error components are affected by commanded velocity, friction magnitude dc, other parameters, k etc

2 d) value from part 1f (following error during constant velocity motion)

The average value of Coulomb friction was found to be:

* Explain methodology in the calculations

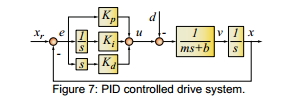
The theoretical value of steady state tracking error in the absence of friction was found to be as shown in 2 c):

2 e) rerun with part trajectory from part 1f with friction, overlay sim and exp tracking results (similar to 1g). [I overlayed them with friction but it’s too hard to tell the difference from 1g because it’s too zoomed out. I might add a zoomed in view at the bottom]

* comment on similarity and discrepancy.

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## Part 3



3(a)

Let

and

by inspection

Isolate for x

3(b)

Steady State

3(c) Routh’s Stability Criterion for

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| --- | --- | --- | --- |
|  | 1 |  | 0 |
|  |  |  | 0 |
|  |  | 0 |  |
|  |  | 0 |  |

3(d) Substituting   
 ,, =1.4705

Solving for upper bound of ki:

3 e) How error changes as you modify K? show in graphs similar to part 1g (use 3 diff cases for kp)

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3 f) Show simulation and experimental results, plot similar to 1g.

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3 g) overlay experimental from parts 1f and 3f. comment on contribution of integral action.

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* Does integral control provide better tracking and disturbance rejection?
* What are the limitations
* Refer to 2c, 3b, 3c to back up comments

3 h) plot pole-zero maps of PD and PID transfer functions on the same graph (use diff symbols for each)

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* Comment on pole and zero locations (refer to natural freq and damping ratio values)

By referring to the plot and using Matlab functions: damp() and zero(), the natural frequencies, damping ratios, poles and zeroes were found to be the following:

|  |  |  |  |
| --- | --- | --- | --- |
| **PD Controller** | | | |
| *Freq* | *Zeta* | *Poles* | *Zeroes* |
| 50.265 | 0.8 | -40.212 ± 30.159i | -36.585 |
| **PID Controller** | | | |
| *Freq* | *Zeta* | *Poles* | *Zeroes* |
| 34.489 | 0.340 | -11.741 ± 32.429i | -18.293 ± 25.420i |
| 56.943 | 1 | -56.943 |

* How are closed loop pole locations affected by the integral?